Transmission of the maximum number of signals through a multiconductor transmission line without crosstalk or return loss: theory and simulation

Vladimir Vulfin

ANSYS inc. 26 Harokmim St., Holon, Israel

Reuven Ianconescu

Shenkar College of Engineering and Design 12, Anna Frank St., Ramat Gan, Israel[*]

Multiconductor communication is widely used in electronic applications for fast data transfer between different devices. The most common multiconductor transmission media are Cat 6 Ethernet cables, flat cables and backplanes. In all applications, the data is transferred differentially between pairs of conductors, so that roughly the number of transmitted signals is half the numbers of conductors. Physically, between N+1 conductors it is possible to transmit N signals and the reason for not doing so is the huge crosstalk and return loss that would arise when transmitting so many signals. The current work suggests an algorithm for transmitting the maximum number of possible signals without crosstalk or return loss.

[*]Electronic address: `riancon@gmail.com,riancon@shenkar.ac.il`

I. INTRODUCTION

In all the applications of fast data transmission via multiconductor transmission lines (MTL), like flat cables, backplanes, etc., the signals are transmitted differentially between pairs of conductors, so that the number of transmitted signals is half the number of conductors. Under those conditions, the manufacturers specify the crosstalk between signals and the matching loads.

One remarks that MTL cannot be matched exactly by individual loads between signals [1–5], unless the differential lines would be mutually isolated. The typical crosstalk of around -20 dB [9] shows that they are not isolated, and hence some return loss must exist.

In addition, we remark that between $N + 1$ conductors one can transfer N independent signals, which means that for large N, the data transfer applications transmit about half of the maximum possible number of signals, hence half of the data rate. Clearly, if one would transfer the full number of possible signals, the crosstalk (and the return loss) would increase substantially.

It is to be mentioned that the software applications we use become more and more complex and require more powerful computers, which require faster data transfers, and we may observe how we are periodically offered higher and higher internet speed. Hence increasing the number of transferred signals to the maximum is an issue of interest.

The current work suggests a method to transfer the maximum number of possible signals, i.e. N signals on MTL of $N + 1$ conductors without crosstalk or return loss. This is achieved by using a pre-processing unit at the input of the transmission line and a post-processing unit at the line output. Certainly, the crosstalk always exists on the MTL itself, but the whole system including the MTL and the processing units is crosstalk free. We shall explain the theory and show simulation results. In this work we consider lossless lines, but the method can be generalized to lossy lines with some degradation.

The work is organized as follows. The first sections explain the theory: in section II we give the description of a MTL by scattering (S) matrix, and the properties of this matrix and in section III we explain how to derive a generalized ABCD matrix from the S matrix. In section IV we show the ABCD description of the MTL and the properties of the generalized ABCD matrix, the connection to the characteristic impedance matrix of the line, and how the system simplifies for homogeneous media. In section V we show how to get the S matrix

from the generalized ABCD matrix, and in section VI we explain the general scheme for a crosstalk and return loss free system. In section VII we show simulation examples of crosstalk and return loss free transmissions: we present one example in homogeneous media and one example in non-homogeneous media. We make use in this section of the theoretical background from the previous sections.

The work is ended by some concluding remarks.

Note: through this work, the unit matrix is noted by $\boldsymbol{U}$, to avoid confusion with current vectors $\boldsymbol{I}$. All the matrix transpose operations are transpose and no conjugate.

II. DESCRIPTION OF THE MTL BY SCATTERING MATRIX

We shall define the scattering matrix of the MTL for equal port impedances R. The near end ports are numbered 1,2, .. N and the far end ports are numbered $N+1$, $N+2$, .. $2N$. The forward voltages into the ports are V_1^+, V_2^+, .. V_N^+, V_{N+1}^+, V_{N+2}^+, .. V_{2N}^+ and will be grouped as $\boldsymbol{V}_{ne}^+$ at the near end and $\boldsymbol{V}_{fe}^+$ at the far end. We use the same grouping for the backward voltages: $\boldsymbol{V}_{ne}^-$ and $\boldsymbol{V}_{fe}^-$.

The scattering matrix $\boldsymbol{S}$ describes the connection between outgoing voltage waves and incoming voltage waves [5–8], provided all port impedances equal each other, which is our case. We group it into 4 submatrices $\boldsymbol{S}_{nn}$ (near to near), $\boldsymbol{S}_{fn}$ (far to near) and so on, $\boldsymbol{S}_{nf}$ and $\boldsymbol{S}_{ff}$, which satisfy

$$\left(\frac{\boldsymbol{V}_{ne}^-}{\boldsymbol{V}_{fe}^-} \right) = \left(\begin{array}{c|c} \boldsymbol{S}_{nn} & \boldsymbol{S}_{fn} \\ \hline \boldsymbol{S}_{nf} & \boldsymbol{S}_{ff} \end{array} \right) \left(\frac{\boldsymbol{V}_{ne}^+}{\boldsymbol{V}_{fe}^+} \right), \tag{1}$$

We may write the following properties for the $\boldsymbol{S}$ matrix: (a) for any reciprocal network S is symmetric so that $\boldsymbol{S}_{nn} = \boldsymbol{S}_{nn}^T$ and $\boldsymbol{S}_{ff} = \boldsymbol{S}_{ff}^T$ and $\boldsymbol{S}_{nf} = \boldsymbol{S}_{fn}^T$. (b) defining indices i and j both between 1 and N, because of the near-far symmetry, $S_{i,j} = S_{N+i,N+j}$, so that $\boldsymbol{S}_{nn} = \boldsymbol{S}_{ff}$, and $S_{i,N+j} = S_{N+i,j}$, so that $\boldsymbol{S}_{fn} = \boldsymbol{S}_{nf}$.

Combining those properties, we define the reflection matrix $\boldsymbol{\Gamma} \equiv \boldsymbol{S}_{nn} = \boldsymbol{S}_{ff}$, which is symmetric, and the transmission matrix $\boldsymbol{\tau} \equiv \boldsymbol{S}_{fn} = \boldsymbol{S}_{nf}$ which is also symmetric. We therefore can rewrite Eq. (1)

$$\left(\frac{\boldsymbol{V}_{ne}^-}{\boldsymbol{V}_{fe}^-} \right) = \left(\begin{array}{c|c} \boldsymbol{\Gamma} & \boldsymbol{\tau} \\ \hline \boldsymbol{\tau} & \boldsymbol{\Gamma} \end{array} \right) \left(\frac{\boldsymbol{V}_{ne}^+}{\boldsymbol{V}_{fe}^+} \right), \tag{2}$$

where the following properties exist

$$\boldsymbol{\Gamma} = \boldsymbol{\Gamma}^T \tag{3}$$

and

$$\boldsymbol{\tau} = \boldsymbol{\tau}^T \tag{4}$$

In the next section we shall derive a generalized ABCD matrix from the scattering matrix.

III.   DERIVATION OF THE GENERALIZED ABCD MATRIX FROM THE SCATTERING MATRIX

Writing the forward (entering) and backward (exiting) voltages in terms of total voltages and currents, we have

$$\boldsymbol{V}^+ = \frac{1}{2}(\boldsymbol{V} + R\boldsymbol{I}) \tag{5}$$

and

$$\boldsymbol{V}^- = \frac{1}{2}(\boldsymbol{V} - R\boldsymbol{I}), \tag{6}$$

where R is the port impedances of the S matrix. Setting Eqs. (5) and (6) into Eq. (2), $\frac{1}{2}$ cancels and we obtain

$$\begin{pmatrix} \boldsymbol{V}_{ne} - R\boldsymbol{I}_{ne} \\ \boldsymbol{V}_{fe} - R\boldsymbol{I}_{fe} \end{pmatrix} = \left(\begin{array}{c|c} \boldsymbol{\Gamma} & \boldsymbol{\tau} \\ \hline \boldsymbol{\tau} & \boldsymbol{\Gamma} \end{array} \right) \begin{pmatrix} \boldsymbol{V}_{ne} + R\boldsymbol{I}_{ne} \\ \boldsymbol{V}_{fe} + R\boldsymbol{I}_{fe} \end{pmatrix}. \tag{7}$$

Opening, rearranging and putting all variables in the left side, results in a 4×2 block matrix of $N \times N$ blocks multiplying a vector of the variables $\boldsymbol{V}_{ne}$, $\boldsymbol{I}_{ne}$, $\boldsymbol{V}_{fe}$ and $-\boldsymbol{I}_{fe}$, equal 0 on the right side. This can be opened into

$$\left(\begin{array}{c|c} \boldsymbol{U} - \boldsymbol{\Gamma} & -R(\boldsymbol{U} + \boldsymbol{\Gamma}) \\ \hline -\boldsymbol{\tau} & -R\boldsymbol{\tau} \end{array} \right) \begin{pmatrix} \boldsymbol{V}_{ne} \\ \boldsymbol{I}_{ne} \end{pmatrix} + $$
$$\left(\begin{array}{c|c} \boldsymbol{\tau} & -R\boldsymbol{\tau} \\ \hline \boldsymbol{U} - \boldsymbol{\Gamma} & R(\boldsymbol{U} - \boldsymbol{\Gamma}) \end{array} \right) \begin{pmatrix} \boldsymbol{V}_{fe} \\ -\boldsymbol{I}_{fe} \end{pmatrix} = 0, \tag{8}$$

from which we can express the near end (voltage and current) in terms of the far end elements, as follows

$$\left(\begin{array}{c} \boldsymbol{V}_{ne} \\ \boldsymbol{I}_{ne} \end{array}\right) = -\left(\begin{array}{c|c} \boldsymbol{U} - \boldsymbol{\Gamma} & -R(\boldsymbol{U} + \boldsymbol{\Gamma}) \\ \hline -\boldsymbol{\tau} & -R\boldsymbol{\tau} \end{array}\right)^{-1}$$

$$\left(\begin{array}{c|c} -\boldsymbol{\tau} & R\boldsymbol{\tau} \\ \hline \boldsymbol{U} - \boldsymbol{\Gamma} & R(\boldsymbol{U} - \boldsymbol{\Gamma}) \end{array}\right) \left(\begin{array}{c} \boldsymbol{V}_{fe} \\ -\boldsymbol{I}_{fe} \end{array}\right) = 0, \tag{9}$$

We define the ABCD matrix by:

$$\left(\begin{array}{c} \boldsymbol{V}_{ne} \\ \boldsymbol{I}_{ne} \end{array}\right) = \left(\begin{array}{c|c} \boldsymbol{A} & \boldsymbol{B} \\ \hline \boldsymbol{C} & \boldsymbol{D} \end{array}\right) \left(\begin{array}{c} \boldsymbol{V}_{fe} \\ -\boldsymbol{I}_{fe} \end{array}\right), \tag{10}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are $N \times N$ matrices and we defined the far end current $\boldsymbol{I}_{fe}$ with opposite sign than this used for other descriptions, like for the $\boldsymbol{S}$ matrix [5–8]. We may invert the matrix in Eq. (9), using the Schur complements, the fact that $\boldsymbol{U} + \boldsymbol{\Gamma}$ commutes with $(\boldsymbol{U} - \boldsymbol{\Gamma})^{-1}$ and the identity $(\boldsymbol{U} + P)^{-1} = \boldsymbol{U} - (\boldsymbol{U} + P)^{-1}P$, obtaining

$$\frac{1}{2}\left(\begin{array}{c|c} \boldsymbol{U} & -(\boldsymbol{U} + \boldsymbol{\Gamma})\boldsymbol{\tau}^{-1} \\ \hline -\boldsymbol{U}/R & -(\boldsymbol{U} - \boldsymbol{\Gamma})\boldsymbol{\tau}^{-1}/R \end{array}\right), \tag{11}$$

and by doing the matrix multiplication in Eq. (9) and compare with Eq. (10), we obtain the ABCD matrix submatrices in terms of the scattering matrix, as follows

$$\boldsymbol{A} = \frac{1}{2}[\boldsymbol{\tau} + (\boldsymbol{U} + \boldsymbol{\Gamma})\boldsymbol{\tau}^{-1}(\boldsymbol{U} - \boldsymbol{\Gamma})], \tag{12}$$

$$\boldsymbol{B} = \frac{R}{2}[-\boldsymbol{\tau} + (\boldsymbol{U} + \boldsymbol{\Gamma})\boldsymbol{\tau}^{-1}(\boldsymbol{U} + \boldsymbol{\Gamma})], \tag{13}$$

$$\boldsymbol{C} = \frac{1}{2R}[-\boldsymbol{\tau} + (\boldsymbol{U} - \boldsymbol{\Gamma})\boldsymbol{\tau}^{-1}(\boldsymbol{U} - \boldsymbol{\Gamma})], \tag{14}$$

and

$$\boldsymbol{D} = \frac{1}{2}[\boldsymbol{\tau} + (\boldsymbol{U} - \boldsymbol{\Gamma})\boldsymbol{\tau}^{-1}(\boldsymbol{U} + \boldsymbol{\Gamma})], \tag{15}$$

Using properties (3) and (4), we remark that

$$\boldsymbol{A} = \boldsymbol{D}^T \tag{16}$$

IV.   DESCRIPTION OF THE MTL BY A GENERALIZED ABCD MATRIX

In this section we shall call the near end vector of voltages and currents $\boldsymbol{V}_{ne} = \boldsymbol{V}_1$ and $\boldsymbol{I}_{ne} = \boldsymbol{I}_1$ and the far end vector of voltages and currents $\boldsymbol{V}_{fe} = \boldsymbol{V}_2$ and $\boldsymbol{I}_{fe} = -\boldsymbol{I}_2$, so that Eq. (10) is more naturally rewritten without a minus sign:

$$\left(\frac{\boldsymbol{V}_1}{\boldsymbol{I}_1} \right) = \left(\begin{array}{c|c} \boldsymbol{A} & \boldsymbol{B} \\ \hline \boldsymbol{C} & \boldsymbol{D} \end{array} \right) \left(\frac{\boldsymbol{V}_2}{\boldsymbol{I}_2} \right), \tag{17}$$

We may rewrite Eq. (17) as

$$\left(\frac{\boldsymbol{V}_1}{-\boldsymbol{I}_1} \right) = \left(\begin{array}{c|c} \boldsymbol{A} & -\boldsymbol{B} \\ \hline -\boldsymbol{C} & \boldsymbol{D} \end{array} \right) \left(\frac{\boldsymbol{V}_2}{-\boldsymbol{I}_2} \right), \tag{18}$$

or

$$\left(\frac{\boldsymbol{V}_2}{-\boldsymbol{I}_2} \right) = \left(\begin{array}{c|c} \boldsymbol{A} & -\boldsymbol{B} \\ \hline -\boldsymbol{C} & \boldsymbol{D} \end{array} \right)^{-1} \left(\frac{\boldsymbol{V}_1}{-\boldsymbol{I}_1} \right), \tag{19}$$

so that the matrix in Eq. (19) describes the same MTL after inverting between near and far end. The inverted system is identical to the original system and therefore its ABCD matrix must be identical

$$\left(\begin{array}{c|c} \boldsymbol{A} & \boldsymbol{B} \\ \hline \boldsymbol{C} & \boldsymbol{D} \end{array} \right)^{-1} = \left(\begin{array}{c|c} \boldsymbol{A} & -\boldsymbol{B} \\ \hline -\boldsymbol{C} & \boldsymbol{D} \end{array} \right). \tag{20}$$

The inverse of a block matrix can be written as

$$\left(\begin{array}{c|c} \boldsymbol{A} & \boldsymbol{B} \\ \hline \boldsymbol{C} & \boldsymbol{D} \end{array} \right)^{-1} =$$

$$\left(\begin{array}{c|c} \boldsymbol{S}_D^{-1} & -\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{S}_A^{-1} \\ \hline -\boldsymbol{D}^{-1}\boldsymbol{C}\boldsymbol{S}_D^{-1} & \boldsymbol{S}_A^{-1} \end{array} \right), \tag{21}$$

where

$$\boldsymbol{S}_A = \boldsymbol{D} - \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B} \tag{22}$$

and

$$\boldsymbol{S}_D = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{D}^{-1}\boldsymbol{C} \tag{23}$$

are the Schur complements of $\boldsymbol{A}$ and $\boldsymbol{D}$ respectively. Therefore,

$$\boldsymbol{S}_D = \boldsymbol{A}^{-1} \tag{24}$$

which implies

$$\det\left(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{D}^{-1}\boldsymbol{C}\right) = 1/\det\boldsymbol{A}. \tag{25}$$

The MTL must be reciprocal, hence the determinant of the ABCD matrix must be 1. So using the block matrices determinant identity

$$1 = \det\left(\begin{array}{c|c} \boldsymbol{A} & \boldsymbol{B} \\ \hline \boldsymbol{C} & \boldsymbol{D} \end{array}\right) =$$

$$\det\left(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{D}^{-1}\boldsymbol{C}\right)\det\boldsymbol{D} = \det\boldsymbol{D}/\det\boldsymbol{A}, \tag{26}$$

hence

$$\det\boldsymbol{A} = \det\boldsymbol{D}, \tag{27}$$

which is the generalization of the *one dimensional* left-right symmetric case for which $A = D$ and confirms the property in Eq. (16).

A.   Diagonalize the elements of the ABCD matrix

Let us define a transformation on the voltages, $\boldsymbol{V} = \boldsymbol{T}_V\boldsymbol{V}'$ and on the currents $\boldsymbol{I} = \boldsymbol{T}_I\boldsymbol{I}'$. Requiring the transformation to be transformer-like implies that $\boldsymbol{V}^T\boldsymbol{I} = \boldsymbol{V}'^T\boldsymbol{I}'$, so that

$$\boldsymbol{V}^T\boldsymbol{I} = \boldsymbol{V}'^T\boldsymbol{T}_V^T\boldsymbol{T}_I\boldsymbol{I}', \tag{28}$$

which must hold for any voltages and currents, so that

$$\boldsymbol{T}_V^T\boldsymbol{T}_I = \boldsymbol{U}. \tag{29}$$

Let us choose the transformation so that it diagonalizes $\boldsymbol{A}$, having

$$\boldsymbol{A}' = \boldsymbol{T}_V^{-1}\boldsymbol{A}\boldsymbol{T}_V, \tag{30}$$

so that $\boldsymbol{A}'$ is the diagonal eigenvalues matrix of $\boldsymbol{A}$. Using Eq. (29) we may also write this as $\boldsymbol{A}' = \boldsymbol{T}_I^T\boldsymbol{A}\boldsymbol{T}_V$. From Eq. (16), which is also confirmed by Eq. (27), we know that the eigenvalues of $\boldsymbol{D}$ are the same as those of $\boldsymbol{A}$. So transposing Eq. (30) we obtain

$$\boldsymbol{D}' = \boldsymbol{A}' = \boldsymbol{A}'^T = \boldsymbol{T}_V^T\boldsymbol{D}\boldsymbol{T}_I. \tag{31}$$

From Eq. (24), we see that $\boldsymbol{S}_D^{-1}$, and hence $\boldsymbol{S}_D$, gets diagonalized by the same transformation which diagonalizes $\boldsymbol{A}$. Let us call $\boldsymbol{S}_D'$ the eigenvalues matrix of $\boldsymbol{S}_D$, and we obtain

$$\boldsymbol{S}_D' = \boldsymbol{T}_I^T\boldsymbol{S}_D\boldsymbol{T}_V \tag{32}$$

Using Eq. (23), and then Eqs. (30) and (31) we get

$$
\begin{aligned}
S'_D &= T_I^T(A - BD^{-1}C)T_V = \\
&\quad A' - T_I^T BD^{-1}CT_V = \\
&\quad A' - T_I^T BT_I D^{-1'}T_V^T CT_V,
\end{aligned}
\tag{33}
$$

from which result the diagonalization of B and C, as follows

$$
B' = T_I^T BT_I
\tag{34}
$$

and

$$
C' = T_V^T CT_V.
\tag{35}
$$

For a matched MTL $V = Z_0 I$, where Z_0 is the characteristic impedance matrix of the MTL. Using the diagonalizing transformations, we have for a matched line

$$
T_V V' = Z_0 T_I I',
\tag{36}
$$

or

$$
V' = Z'_0 I',
\tag{37}
$$

where

$$
Z'_0 = T_I^T Z_0 T_I,
\tag{38}
$$

is the diagonal eigenvalues matrix of the characteristic impedance matrix. Each element of V' and I', represents a *mode*.

B. Obtaining the characteristic impedance matrix from the generalized ABCD matrix

When the MTL is matched, the relation between the voltage vector V and the current vector I is $V = Z_0 I$ anywhere on the transmission line, and specifically at the input and output of the line. In such case Eq. (17) may be written

$$
\left(\begin{array}{c} Z_0 I_1 \\ \hline I_1 \end{array} \right) = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right) \left(\begin{array}{c} Z_0 I_2 \\ \hline I_2 \end{array} \right),
\tag{39}
$$

which may be rearranged into

$$\left(\begin{array}{c|c} \boldsymbol{Z}_0 & -(\boldsymbol{A}\boldsymbol{Z}_0 + \boldsymbol{B}) \\ \hline \boldsymbol{U} & -(\boldsymbol{C}\boldsymbol{Z}_0 + \boldsymbol{D}) \end{array}\right) \left(\begin{array}{c} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{array}\right) = 0. \tag{40}$$

For non zero currents, the determinant of the above matrix must be 0. Because the $\boldsymbol{U}$ term, one may easily express the determinant, getting a square matrix equation for $\boldsymbol{Z}_0$

$$\boldsymbol{Z}_0 \boldsymbol{C} \boldsymbol{Z}_0 + \boldsymbol{Z}_0 \boldsymbol{D} - \boldsymbol{A} \boldsymbol{Z}_0 - \boldsymbol{B} = 0 \tag{41}$$

Multiplying from left by $\boldsymbol{T}_I^T$ and from right by $\boldsymbol{T}_I$ and using Eqs. (30)-(35) and Eq. (38), all the matrices in Eq. (41) diagonalize and we are left with a regular square equation for the *individual* eigenvalues

$$Z_0'^2 C' + Z_0'(D' - A') - B' = 0, \tag{42}$$

and using $D' = A'$, the eigenvalues of $\boldsymbol{Z}_0$ are

$$Z_0' = \sqrt{B'/C'}, \tag{43}$$

where we take the plus sign for the forward moving waves. The input-output relation for *each* mode on a matched line is obtained via the ABCD matrix for this mode, as follows

$$\left(\begin{array}{c} Z_0' I_1' \\ I_1' \end{array}\right) = \left(\begin{array}{cc} A' & B' \\ C' & D' \end{array}\right) \left(\begin{array}{c} Z_0' I_2' \\ I_2' \end{array}\right), \tag{44}$$

where for brevity we do not write the index of the mode, understanding that non bold values represent values belonging to a mode. This results in

$$I_1' = (A' + B'/Z_0') I_2'. \tag{45}$$

Using Eq. (43), $D' = A'$ and the fact that the determinant of the single mode ABCD matrix is 1, one obtains

$$I_1' = \left(A' + \sqrt{A'^2 - 1}\right) I_2'. \tag{46}$$

One usually defines in such case $A' \equiv \cosh(\gamma)$, so that $\left(A' + \sqrt{A'^2 - 1}\right) = \exp(\gamma)$, but being rather interested in lossless propagating waves we define

$$A' \equiv \cos(\theta), \tag{47}$$

obtaining

$$I_1' = \exp(j\theta) I_2', \tag{48}$$

so that θ is the propagation shift angle of the given mode i for a matched line (losses can still be incorporated by letting θ be complex). One can calculate the relative (equivalent) dielectric constant ϵ_{eq} for the given mode by the relation

$$\theta = \frac{2\pi f l}{c}\sqrt{\epsilon_{eq}}, \tag{49}$$

where f is the frequency, l is the MTL length and c is the speed of light in vacuum.

C. Simplification for homogeneous media

In [2] it is shown that for a MTL in homogeneous medium, one can express the voltage-current vector pair at any location on the line as function of the voltage-current vector pair at another location. Specifically, between near and far end

$$\begin{pmatrix} \boldsymbol{V}_1 \\ \boldsymbol{I}_1 \end{pmatrix} = \left(\begin{array}{c|c} \boldsymbol{U}\cos(\theta) & j\boldsymbol{Z}_0\sin(\theta) \\ \hline j\boldsymbol{Y}_0\sin(\theta) & \boldsymbol{U}\cos(\theta) \end{array} \right) \begin{pmatrix} \boldsymbol{V}_2 \\ \boldsymbol{I}_2 \end{pmatrix}, \tag{50}$$

where $\boldsymbol{Y}_0 = \boldsymbol{Z}_0^{-1}$ is the characteristic admittance matrix and

$$\theta = \frac{2\pi f l}{c}\sqrt{\epsilon_r} \tag{51}$$

is the propagation shift angle of the whole voltage-current wave. In this case $\boldsymbol{A} = \boldsymbol{D} = \boldsymbol{U}\cos(\theta)$ is diagonal with equal elements, so that any orthogonal matrix diagonalizes $\boldsymbol{A}$ or $\boldsymbol{D}$, hence $\boldsymbol{T}_V = \boldsymbol{T}_I \equiv \boldsymbol{T}$. The matrix $\boldsymbol{T}$ diagonalizes the characteristic impedance matrix $\boldsymbol{Z}_0$, which is a symmetric matrix, so that

$$\boldsymbol{Z}_0' = \boldsymbol{T}^T \boldsymbol{Z}_0 T, \tag{52}$$

V. DERIVATION OF THE SCATTERING MATRIX FROM A GENERALIZED ABCD MATRIX

Starting from Eq. (10), we express the voltages and the current vectors in terms of entering and exiting waves voltage vectors

$$\boldsymbol{V} = \boldsymbol{V}^+ + \boldsymbol{V}^- \tag{53}$$

and

$$R\boldsymbol{I} = \boldsymbol{V}^+ - \boldsymbol{V}^- \tag{54}$$

where R is the port impedances of the S matrix. Using those for the near and far ends in Eq. (10), we get

$$\begin{pmatrix} \mathbf{V}_{ne}^{+} + \mathbf{V}_{ne}^{-} \\ (\mathbf{V}_{ne}^{+} - \mathbf{V}_{ne}^{-})/R \end{pmatrix} = \left(\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & \mathbf{D} \end{array} \right) \begin{pmatrix} \mathbf{V}_{fe}^{+} + \mathbf{V}_{fe}^{-} \\ -(\mathbf{V}_{fe}^{+} - \mathbf{V}_{fe}^{-})/R \end{pmatrix}. \tag{55}$$

Opening, rearranging and putting all variables in the left side, results in a 4×2 block matrix of $N \times N$ blocks multiplying a vector of the variables $\mathbf{V}_{ne}^{-}$, $\mathbf{V}_{fe}^{-}$, $\mathbf{V}_{ne}^{+}$ and $\mathbf{V}_{fe}^{+}$, equal 0 on the right side, which can be rearranged in

$$\left(\begin{array}{c|c} \mathbf{U} & -(\mathbf{A} + \mathbf{B}/R) \\ \hline -\mathbf{U} & -(R\mathbf{C} + \mathbf{D}) \end{array} \right) \begin{pmatrix} \mathbf{V}_{ne}^{-} \\ \mathbf{V}_{fe}^{-} \end{pmatrix} + \left(\begin{array}{c|c} \mathbf{U} & -(\mathbf{A} - \mathbf{B}/R) \\ \hline \mathbf{U} & -(R\mathbf{C} - \mathbf{D}) \end{array} \right) \begin{pmatrix} \mathbf{V}_{ne}^{+} \\ \mathbf{V}_{fe}^{+} \end{pmatrix} = 0, \tag{56}$$

from which we can express the exiting voltage vector waves in terms of the entering voltage vector waves, as follows

$$\begin{pmatrix} \mathbf{V}_{ne}^{-} \\ \mathbf{V}_{fe}^{-} \end{pmatrix} = - \left(\begin{array}{c|c} \mathbf{U} & -(\mathbf{A} + \mathbf{B}/R) \\ \hline -\mathbf{U} & -(R\mathbf{C} + \mathbf{D}) \end{array} \right)^{-1} \left(\begin{array}{c|c} \mathbf{U} & -(\mathbf{A} - \mathbf{B}/R) \\ \hline \mathbf{U} & -(R\mathbf{C} - \mathbf{D}) \end{array} \right) \begin{pmatrix} \mathbf{V}_{ne}^{+} \\ \mathbf{V}_{fe}^{+} \end{pmatrix} = 0, \tag{57}$$

so that the above connection matrix is the scattering matrix. We may invert the matrix in Eq. (57), using the Schur complements and perform the matrix multiplication to obtain the $\mathbf{S}$ matrix block components

$$\boldsymbol{\tau} = 2[\mathbf{A} + \mathbf{B}/R + R\mathbf{C} + \mathbf{D}]^{-1}. \tag{58}$$

and

$$\boldsymbol{\Gamma} = -\frac{1}{2}\boldsymbol{\tau}[\mathbf{A} - \mathbf{B}/R + R\mathbf{C} - \mathbf{D}] \tag{59}$$

One may set $\boldsymbol{\tau}$ and $\boldsymbol{\Gamma}$ from Eqs. (58) and (59) into Eqs. (12)-(15) to confirm their validity.

VI. SCHEME FOR A MATCHED AND CROSSTALK FREE TRANSMISSION

We introduce a pre-processing transformation $\boldsymbol{G}_{pre}$ at the input of the MTL and a post-processing transformation $\boldsymbol{G}_{post}$ at the output of the MTL, as illustrated in Figure 1. The
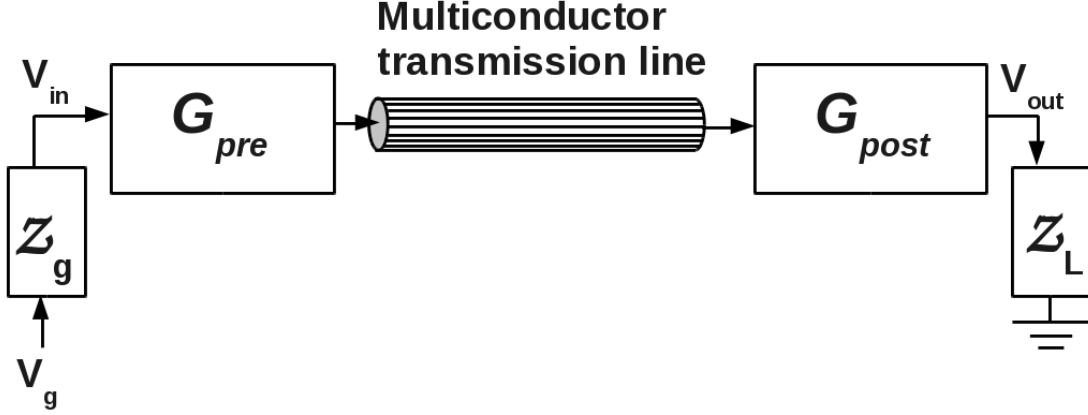

FIG. 1: Multiconductor communication implementing a matched and crosstalk free transmission scheme. A pre-processing transformation unit is implemented at the input of the MTL and a post-processing transformation unit is implemented at the output of the line.

line is fed by the maximum number of independent generator signals $\boldsymbol{V}_{gi}$, for $i = 1, 2..N$, each having the internal impedance Z_{gi} and loaded at the far end by the loads Z_{Li}, so that the generator and load impedances are described by diagonal matrices Z_g and Z_L. In general, Z_g and Z_L may be complex, but matching a complex load to transmission line requires more sophistication even in the one dimensional case, so for the purpose of this work, Z_g and Z_L are considered real.

Considering a multidimensional linear transformation, between N input voltages $\boldsymbol{V}_1$ and N output voltages $\boldsymbol{V}_2$, so that

$$\boldsymbol{V}_2 = \boldsymbol{G}\boldsymbol{V}_1 \tag{60}$$

A transformer-like transformation satisfies $\boldsymbol{V}_1^T \boldsymbol{I}_1 = \boldsymbol{V}_2^T \boldsymbol{I}_2$ and combining this with Eq. (60) we obtain $\boldsymbol{V}_1^T \boldsymbol{I}_1 = \boldsymbol{V}_1^T \boldsymbol{G}^T \boldsymbol{I}_2$, which must hold for any $\boldsymbol{V}_1$, hence

$$\boldsymbol{I}_1 = \boldsymbol{G}^T \boldsymbol{I}_2. \tag{61}$$

Now if the output voltages vector $\boldsymbol{V}_2$ is related to the output currents vector $\boldsymbol{I}_2$ by a load network having an impedance matrix $\boldsymbol{Z}$, so that

$$\boldsymbol{V}_2 = \boldsymbol{Z}\boldsymbol{I}_2, \tag{62}$$

the relation between the input voltages and currents vectors is

$$V_1 = Z_{in}I_1, \tag{63}$$

Z_{in} being the reflected impedance, given by

$$Z_{in} = G^{-1}Z(G^{-1})^T, \tag{64}$$

Which looks like the generalization of a one dimensional transformer. For the MTL to be matched at output one requires

$$Z_0 = G_{post}^{-1}Z_L(G_{post}^{-1})^T. \tag{65}$$

Using Eqs. (38) and (29), one finds that (65) is satisfied by

$$G_{post} = \sqrt{Z_L Z_0'^{-1}}T_V^{-1}. \tag{66}$$

Similarly, we require that looking "back" from the MTL input toward the generator, the Thevénin equivalent impedance matrix be the characteristic impedance matrix Z_0, implying

$$Z_0 = G_{pre}Z_g G_{pre}^T, \tag{67}$$

and again using Eqs. (38) and (29), one finds that (67) is satisfied by

$$G_{pre} = T_V\sqrt{Z_0'Z_g^{-1}}. \tag{68}$$

We remark that both G_{pre} and G_{post} are easily computed because Z_L, Z_g and Z_0' are diagonal matrices.

When using the processing units, one easily obtains the values of V_{in} and V_{out} in Figure 1, as follows

$$V_{in\,(proc)} = \frac{1}{2}V_g, \tag{69}$$

and

$$V_{out\,(proc)} = \sqrt{Z_L Z_g^{-1}}V_{in\,(proc)}\exp(-j\boldsymbol{\theta}), \tag{70}$$

where $\exp(-j\boldsymbol{\theta})$ is a diagonal matrix describing the delays of the modes, see Eq. (49).

We shall need also V_{in} and V_{out} without the processing units. Using the ABCD description of the MTL

$$\begin{pmatrix} V_{in} \\ I_{in} \end{pmatrix} = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right) \begin{pmatrix} V_{out} \\ I_{out} \end{pmatrix}, \tag{71}$$

and $\boldsymbol{V}_{out} = \boldsymbol{Z}_L \boldsymbol{I}_{out}$, we obtain

$$\boldsymbol{V}_{in} = \boldsymbol{Z}_{in} \boldsymbol{I}_{in}, \tag{72}$$

where

$$\boldsymbol{Z}_{in} = (\boldsymbol{A}\boldsymbol{Z}_L + \boldsymbol{B})(\boldsymbol{C}\boldsymbol{Z}_L + \boldsymbol{D})^{-1}, \tag{73}$$

Using $\boldsymbol{V}_g = \boldsymbol{Z}_g \boldsymbol{I}_{in} + \boldsymbol{V}_{in}$, we obtain

$$\boldsymbol{V}_{in\,(no\,proc)} = (\boldsymbol{Z}_g \boldsymbol{Y}_{in} + \boldsymbol{U})^{-1} \boldsymbol{V}_g, \tag{74}$$

where $\boldsymbol{Y}_{in} = \boldsymbol{Z}_{in}^{-1}$. Using $\boldsymbol{V}_{out} = \boldsymbol{Z}_L \boldsymbol{I}_{out}$ we obtain from Eq. (71)

$$\boldsymbol{V}_{out\,(no\,proc)} = (\boldsymbol{A} + \boldsymbol{B}\boldsymbol{Y}_L)^{-1} \boldsymbol{V}_{in\,(no\,proc)}, \tag{75}$$

where $\boldsymbol{Y}_L = \boldsymbol{Z}_L^{-1}$.

VII. SIMULATIONS

We shall use the circuit described in Figure 2 for the simulations. We consider both cases of with or without the pre/post-processing blocks. In the case without pre/post-processing, the blocks are shortened, and each electrical line just passes through.

From the physical point of view, we shall examine both cases of homogeneous and non homogeneous media, i.e. in case the wires in Figure 2 are in free space or surrounded by an insulator, respectively. The last case is non homogeneous, because part of the fields are inside the insulator and part in the surrounding air.

For all cases, the MTL in Figure 2 is fully simulated electromagnetically using HFSS commercial software (FEM method), and modeled into S parameters block, for all the needed frequencies. This S parameters block is further incorporated into ANSYS commercial circuit level simulation tool, named "Designer".

The Designer circuit without the processing units is shown in Figure 3. For the circuit with the processing units, we need to implement the transformations $\boldsymbol{G}_{pre}$ and $\boldsymbol{G}_{post}$. For the purpose of this work, those are implemented by $N \times N$ transformers. Figure 4 shows such an implementation for a 2×2 transformation with coefficients $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

We used all the load impedances equal to the generator impedances, and in this case one remarks from Eqs. (66) and (68) that $\boldsymbol{G}_{post} = \boldsymbol{G}_{pre}^{-1}$, which means changing inputs with
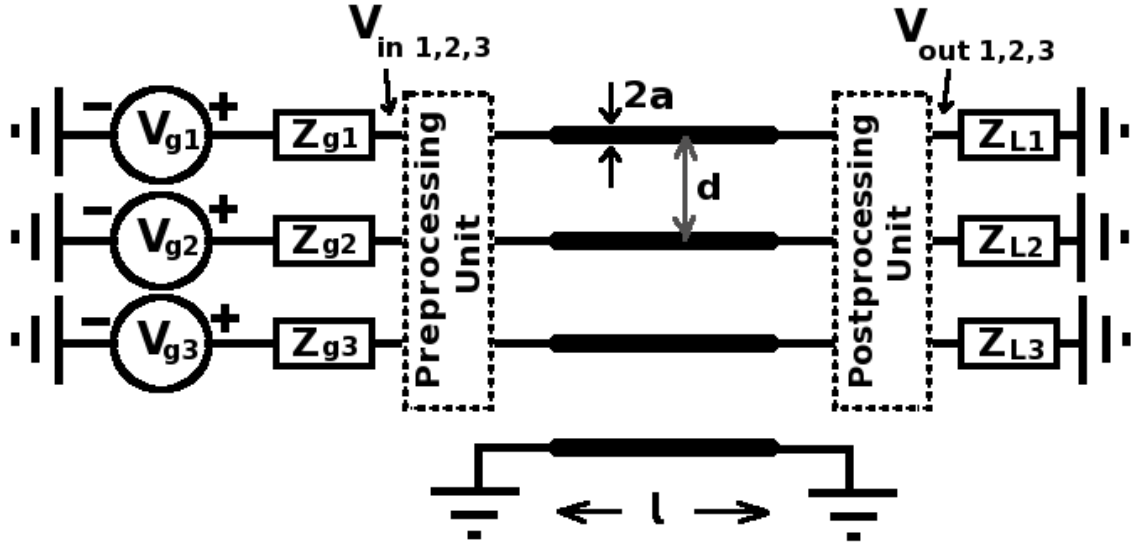
FIG. 2: The geometry is based on flat cables, the conductors are cylindrical with radius $a = 0.1605$ mm in free space, and the distance between their centers is $d = 1.27$ mm. The number of conductors is 4, hence $N = 3$. The MTL length is $l = 20$ cm (the figure is not in proportion due to space limitations). The conductors 1, 2 and 3 are fed by $V_{gi} = 2$V with an internal impedance of $Z_{gi} = 50\Omega$, at 50, 100 and 200MHz respectively (which represent forward voltages of 1V through transmission lines of 50Ω), and are loaded at the far end with $Z_{Li} = 50\Omega$. The conductors are numbered up-down by 1, 2 and 3 and the grounded conductor is the common. The near and far end grounds are of course different electric points and for simplicity we use the same symbol for them. The near and far end lumped elements are referred to the near and far end grounds, respectively.

outputs. This simplifies the Designer circuit which incorporates the processing units, which is shown in Figure 5.

In the following subsections we shall present the simulation results vs the theoretical results for the homogeneous and non homogeneous cases.

A. Homogeneous medium

In this section we consider the conductors shown in Figure 2 in free space, i.e. without insulation, and the simulated geometry is shown in Figure 6. We run the electromagnetic simulation and obtained the scattering matrices for port impedance $R = 50\Omega$ for the 3
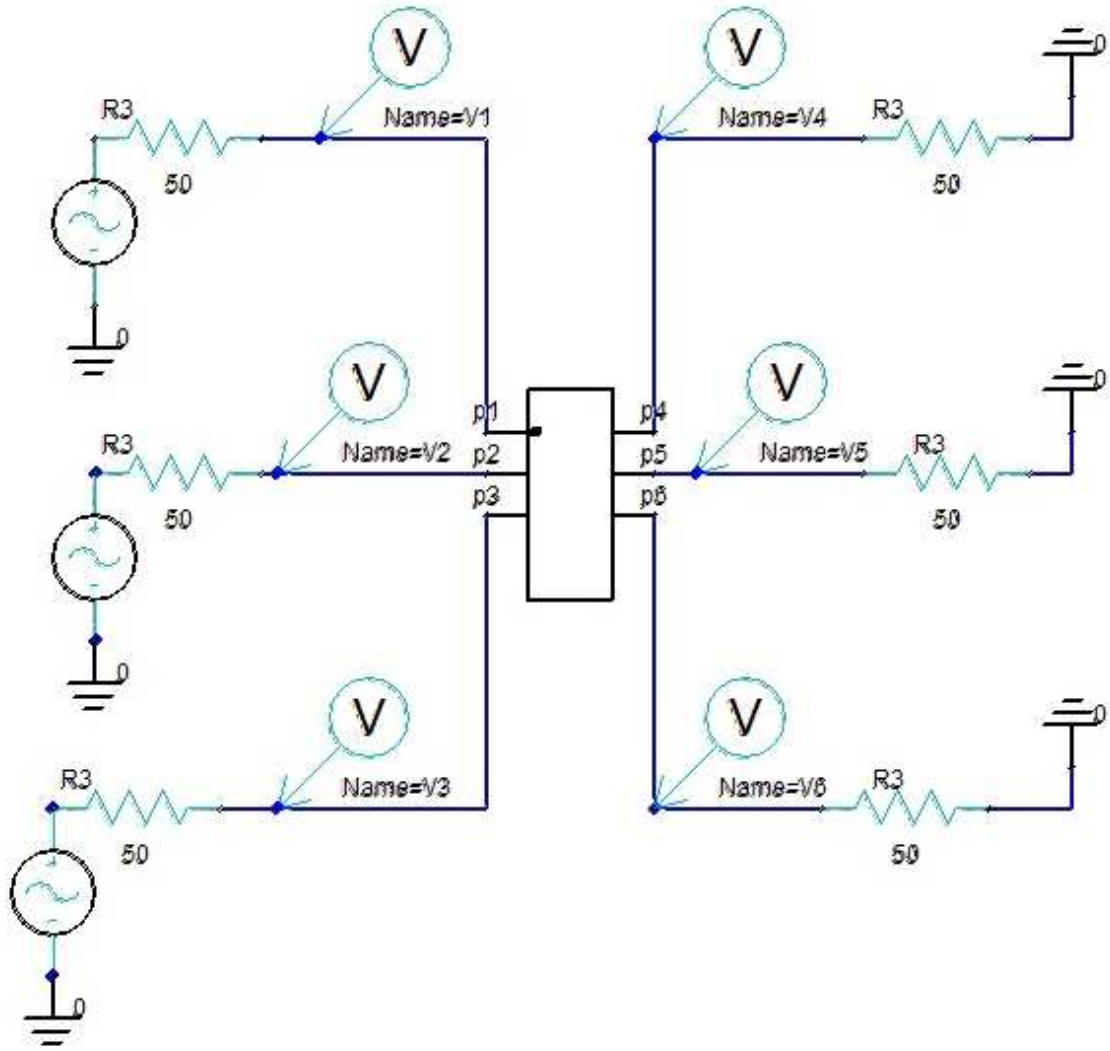
FIG. 3: Designer circuit without pre/post-processing. The middle block represents the S matrices derived for the MTL with ports p1-p3 for the near end and ports p4-p6 for the far end. We measured the voltages at the near end ($\boldsymbol{V}_{in}$) and far end ($\boldsymbol{V}_{out}$) of the MTL.

frequencies: 50, 100 and 200MHz.

On the other hand, the characteristic impedance matrix has been calculated in [2] for the geometry in Figure 2, assuming $a \ll d$ and is given by

$$Z_{0\ ij} = \begin{cases} \frac{\eta_0}{\pi} \ln \frac{d(N+1-j)}{a} & i = j \\ \frac{\eta_0}{2\pi} \ln \frac{d(N+1-j)(N+1-i)}{a|j-i|} & i \neq j \end{cases}, \tag{76}$$

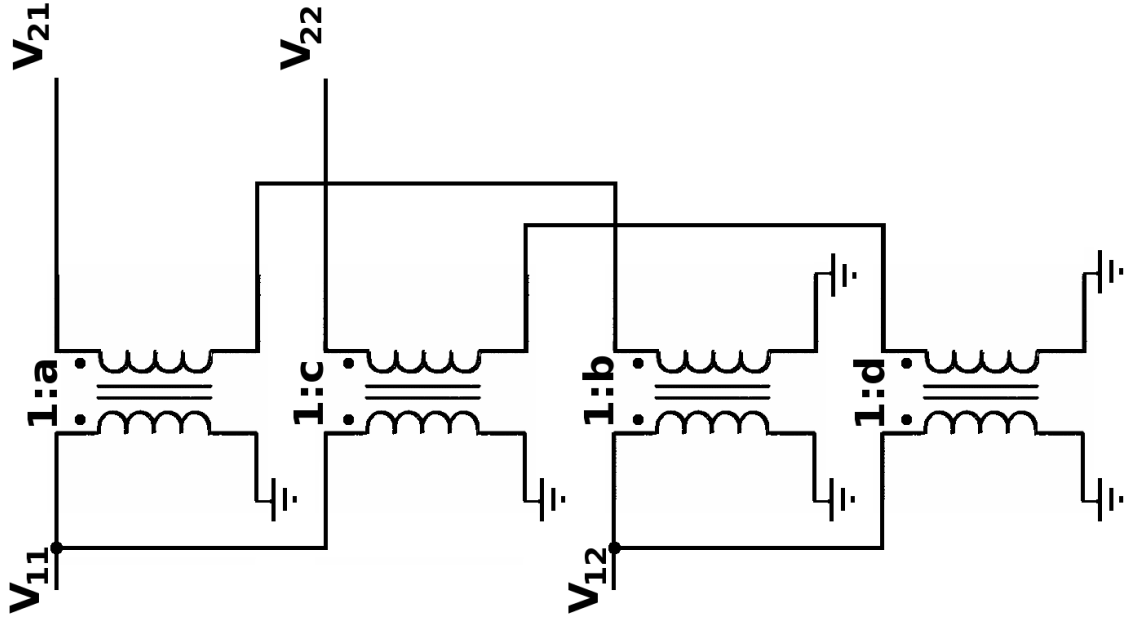where $\eta_0 = 377\Omega$ is the free space impedance. This results for $N = 3$ and the dimensions in

FIG. 4: The implementation of a 2×2 linear transformation between $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$. The primary voltage vector $\boldsymbol{V}_1$ components are V_{11} and V_{12} and the secondary voltage vector $\boldsymbol{V}_2$ components are V_{21} and V_{22}. The transformation implements $V_{21} = aV_{11} + bV_{12}$ and $V_{22} = cV_{11} + dV_{12}$. Because the "Designer" simulation can only implement transformers of positive ratio, a negative coefficient requires interchanging the wires at one side of the individual transformer.

Figure 2 in

$$\boldsymbol{Z}_0 = \frac{\eta_0}{\pi} \begin{pmatrix} 3.1671 & 1.9301 & 1.2370 \\ 1.9301 & 2.7616 & 1.3808 \\ 1.2370 & 1.3808 & 2.0685 \end{pmatrix}. \tag{77}$$

The eigenvalues of this characteristic impedance matrix Z'_0 are: 697.69, 153.91 and 108.06Ω, and we will compare them with the eigenvalues for the non homogeneous case, see next section.

Now using Eqs. (50) and (51) with $\epsilon_r = 1$ we obtain the ABCD matrix for any frequency, and using it in Eqs. (59) and (58) with $R = 50\Omega$ we obtained the S matrix for any frequency and compared them to the S matrices obtained from simulation. The differences between the analytic solution and simulation is around 0.5%, so that the simulation validates the theory in [1–3].

Next we run the Designer simulation without the processing units, as described in Figure 3. The Designer results are compared with analytic results obtained from Eqs. (74) and
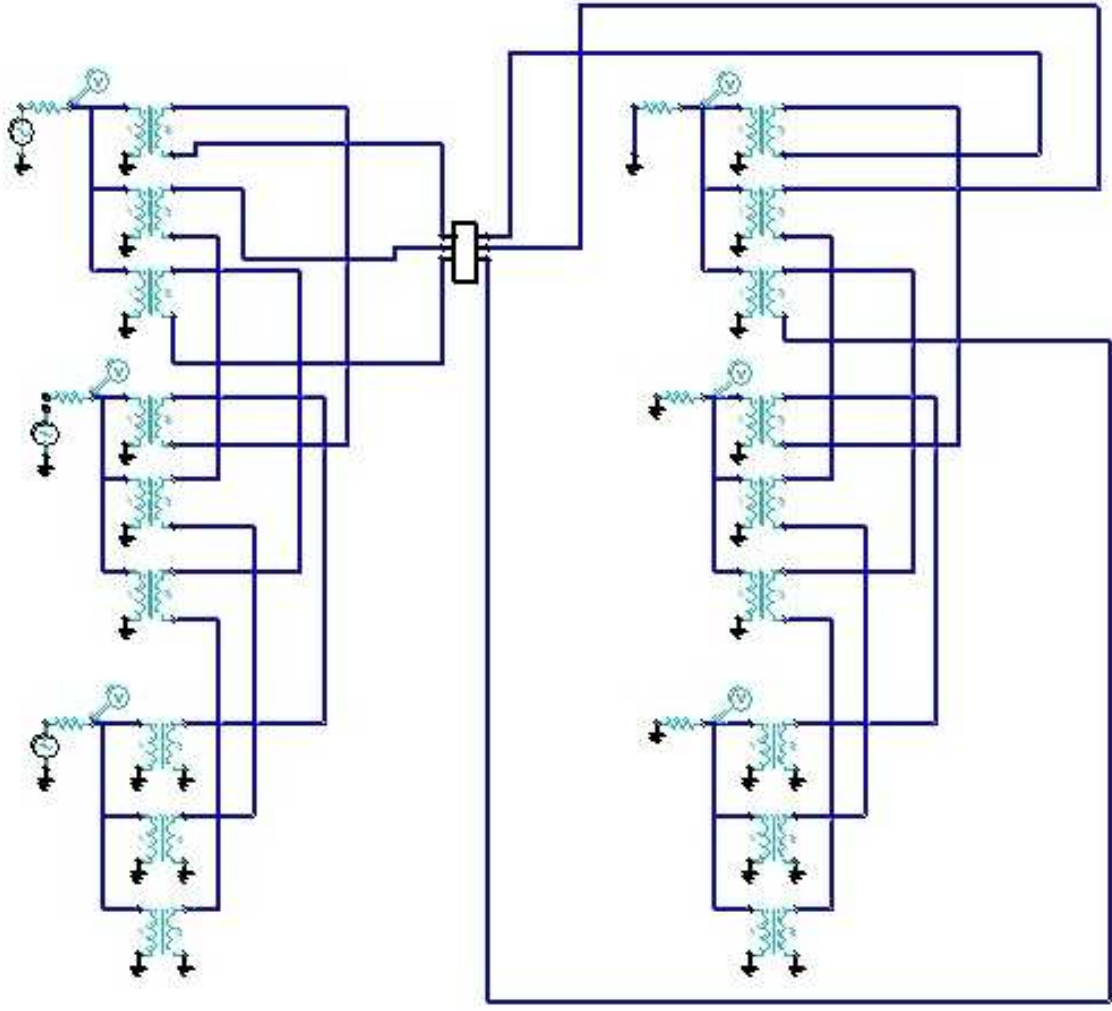
FIG. 5: Designer with pre/post-processing units. The small block represents the S matrices derived for the MTL. We measured the voltages at the input of the pre-processing unit (V_{in}) and at the output of the post-processing unit (V_{out}). It is to be mentioned that the transformers' block is for the homogeneous case below, and we will not supply an additional figure for the non homogeneous case which will come after.

(75) and shown in table I. The simulated results reproduce with a very high accuracy the theoretical results in table I. We remark that there is crosstalk, for example the 100MHz generator is applied only to conductor 2, but at the near end we have 1.47V on conductor 2 and around 0.28V and 0.21V on conductors 1 and 3. So in average the near end crosstalk for conductor 2 is 17%. The system is of course not matched, output voltages differ from input voltages by magnitude and not only by phase.

We calculate now the transformation needed to obtain a perfect match and eliminate the
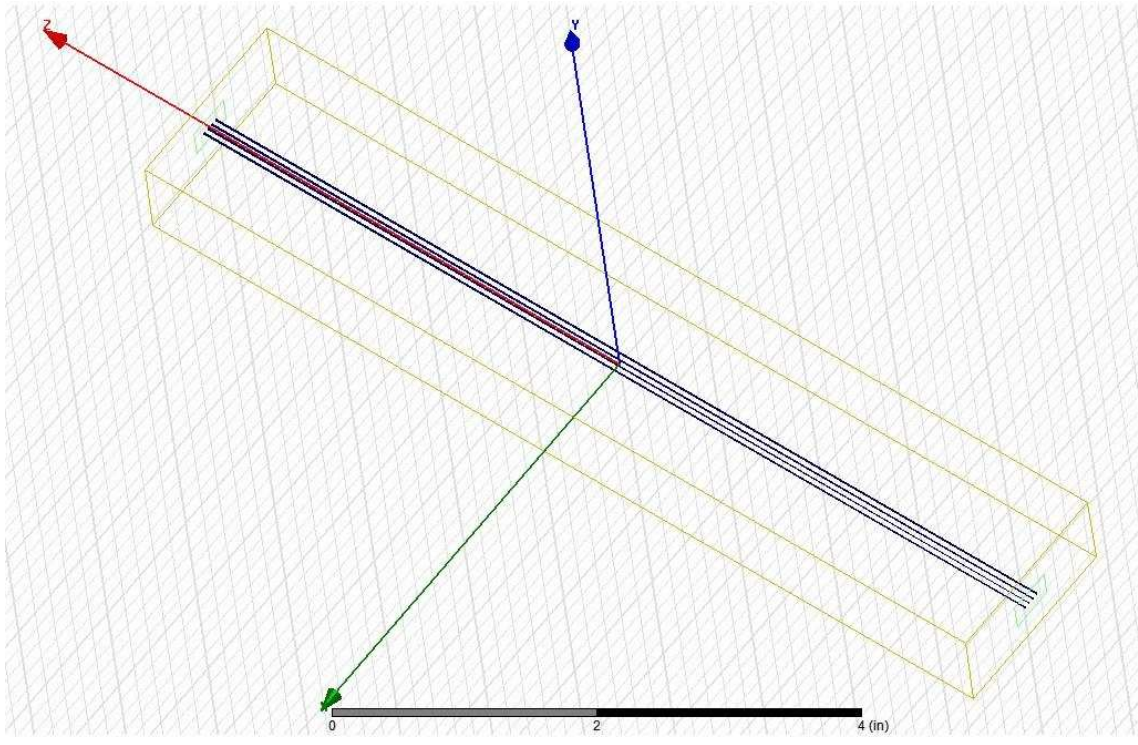
FIG. 6: Conductors in free space simulation geometry.

TABLE I: Voltages V_{in} and V_{out} measured by Designer, and calculated analytically without processing units. Magnitude is in [V], phase is in degrees and the upper row shows the frequencies in [MHz].

	Simulated			Calculated		
f	50	100	200	50	100	200
$V_{in\,1}$	1.38∠13.8	0.28∠-1.45	0.093∠-45.9	1.38∠13.9	0.28∠-1.97	0.091∠-47.3
$V_{in\,2}$	0.27∠23.5	1.47∠11.2	0.175∠-13.7	0.27∠23.1	1.47∠11.2	0.174∠14.6
$V_{in\,3}$	0.17∠14.6	0.21∠4.62	1.67∠9.8	0.17∠13.9	0.22∠3.94	1.67∠9.8
$V_{out\,1}$	0.76∠-30	0.28∠166.1	0.126∠118.6	0.76∠-30	0.28∠165.8	0.123∠118.2
$V_{out\,2}$	0.26∠-162.5	0.69∠-37.3	0.205∠134.1	0.26∠-162.8	0.69∠-37.6	0.205∠133.9
$V_{out\,3}$	0.16∠-168.9	0.21∠170	0.642∠-60.8	0.16∠-169.4	0.21∠170	0.634∠-61

crosstalk. Diagonalizing $\boldsymbol{Z}_0$ (see Eq. (52), we find the matrix T (which is equal to $\boldsymbol{T}_V$ and $\boldsymbol{T}_I$ is this case), and the characteristic impedance matrix eigenvalues $\boldsymbol{Z}_0'$. Using Eq. (68) we

find

$$\boldsymbol{G}_{pre} = \begin{pmatrix} -0.55454 & -1.15029 & 2.44343 \\ 1.13404 & 0.29546 & 2.29228 \\ -0.75339 & 1.29142 & 1.65197 \end{pmatrix}. \tag{78}$$

and $\boldsymbol{G}_{post} = \boldsymbol{G}_{pre}^{-1}$, as explained before.

The $\boldsymbol{G}_{pre}$ matrix is implemented by transformers, and the way to implement it is explained in Figure 4. Figure 4 shows a 2x2 example of such a matrix and one observes the transformers ratios are the columns of the transformation matrix, so that one uses the ratios a and c for first input, b and d for second input (and so on for the general case). Now one may compare the $\boldsymbol{G}_{pre}$ for homogeneous (Eq. 78) with Figure 5 (which is the implementation of the matrix in Eq. 78).

First column: -0.55, 1.13, -0.75. Those are the ratios of the 3 upper transformers in Figure 5. One remarks the wire inversion at the secondary of 1st and 3rd in this group, so that the lower wire connects to the next stage. For non inverted wires, the upper wire gets connected to the next stage (like in the 2nd transformer which implements the positive ratio 1.13).

The Designer simulated results and the analytic results are presented in table II. The

TABLE II: Voltages V_{in} and V_{out} measured by Designer, and calculated analytically with processing units. Magnitude is in [V], phase is in degrees and the upper row shows the frequencies in [MHz].

	Simulated			Calculated		
f	50	100	200	50	100	200
$V_{in\,1}$	1∠-0.23	0.47m∠-111.8	1.7m∠-148.1	1∠0	0	0
$V_{in\,2}$	0.24m∠-100.9	1∠-0.57	1.9m∠-161.1	0	1∠0	0
$V_{in\,3}$	0.47m∠104.5	0.98m∠-126.2	1∠0	0	0	1∠0
$V_{out\,1}$	1∠-12.1	0.3m∠-118	1m∠38.1	1∠-12	0	0
$V_{out\,2}$	0.15m∠-104.2	1∠-24.2	5.6m∠34.2	0	1∠-24	0
$V_{out\,3}$	0.27m∠77.2	2.8m∠62.1	1∠-48.9	0	0	1∠-48

analytic results show exactly $1V\angle 0$, at inputs, i.e. $\frac{1}{2}V_g$, according to Eq. (69) and the same voltage at output (according to Eq. (70) with $\boldsymbol{Z}_g = \boldsymbol{Z}_L$) with the delays given by Eq. (51), which come out 12°, 24° and 48° for the frequencies 50, 100 and 200MHz, respectively. The

results obtained by the Designer simulation are very close to the analytical results. Those differences are due the assumption $a \ll d$ and neglecting the edge effects in the analytic calculations.

B. Non homogeneous medium

In this section we cover the conductors shown in Figure 2 with an insulator of relative dielectric constant $\epsilon_r = 2.1$ (for all frequencies) of dimensions 2mm on 6mm as shown in cross section in Figure 7. We run the electromagnetic simulation and obtained the scattering
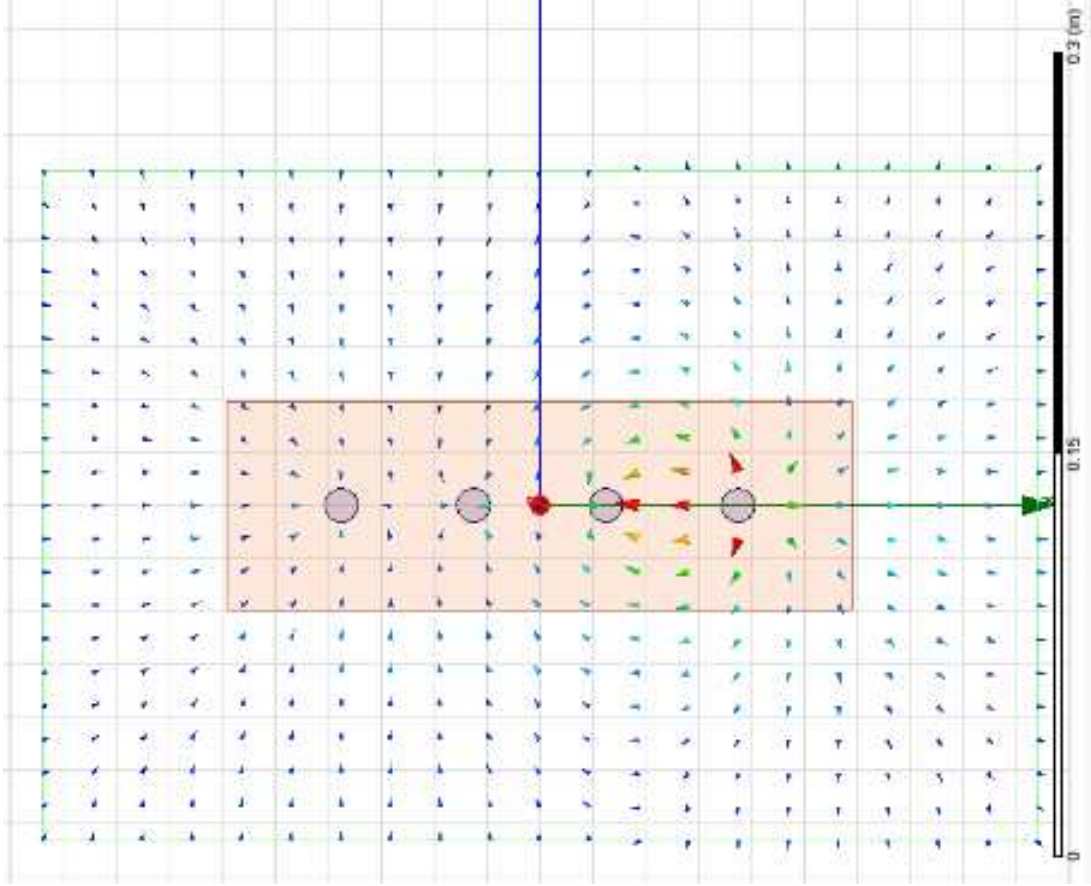
FIG. 7: Cross section of the conductors inside the insulator. The arrows show the electric field for some feeding and one remarks that it is partially in the insulator and partially in the surrounding space. The relative dielectric constant usually changes slowly with frequency, and for simplicity we implemented $\epsilon_r = 2.1$ for the whole range of frequencies we deal with.

matrices for port impedance $R = 50\Omega$ for the 3 frequencies: 50, 100 and 200MHz. In this

case we do not have an analytic solution, so we obtain ABCD matrices from the S matrices using Eqs. (12)-(15), from which we find the analytic solution without processing units from Eqs. (74) and (75).

We run the Designer simulation without the processing units, as described in Figure 3. The Designer results are compared with analytic results and shown in table III. The ana-

TABLE III: Voltages V_{in} and V_{out} measured by Designer, and calculated analytically without processing units, for the non homogeneous case. Magnitude is in [V], phase is in degrees and the upper row shows the frequencies in [MHz].

	Simulated			Calculated		
f	50	100	200	50	100	200
$V_{in\,1}$	1.37∠12.9	0.29∠4.47	0.077∠-36.7	1.37∠12.9	0.29∠4.47	0.077∠-36.7
$V_{in\,2}$	0.28∠26.2	1.44∠8.71	0.21∠-0.51	0.28∠26.2	1.44∠8.71	0.21∠-0.51
$V_{in\,3}$	0.17∠15.5	0.23∠10.7	1.60∠ 5.13	0.17∠15.5	0.23∠10.7	1.60∠5.13
$V_{out\,1}$	0.77∠-31.6	0.28∠158.6	0.14∠112.3	0.77∠-31.6	0.28∠158.6	0.14∠112.3
$V_{out\,2}$	0.25∠-166.1	0.72∠-42.9	0.21∠114.3	0.25∠-166.1	0.72∠-42.9	0.21∠114.3
$V_{out\,3}$	0.16∠-170.2	0.21∠161.6	0.72∠-72.8	0.16∠-170.1	0.21∠161.6	0.72∠-72.8

lytic results are almost identical to the results obtained from the Designer simulation, the differences appearing in less significant digits, which are not shown. This is understandable, because the analytic solutions have been derived from the ABCD matrices, which have been derived from the S matrices obtained by the electromagnetic simulations, used also by the Designer.

Clearly there is crosstalk, for example the 100MHz generator is applied only to conductor 2, but at the near end we have 1.44V on conductor 2 and around 0.29V and 0.23V on conductors 1 and 3. So in average the near end crosstalk for conductor 2 is 18%. The system is of course not matched, output voltages differ from input voltages by magnitude and not only by phase.

Next we diagonalize the $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ elements by Eqs. (30),(31), (34) and (35), obtaining $\boldsymbol{T}_V$, and find the characteristic impedance matrix eigenvalues from Eq. (43).

It is to be mentioned that the above has been computed using the S matrices for the 3 frequencies, and all of them resulted in the same transformation $\boldsymbol{T}_V$ and the same eigenvalues

$\boldsymbol{Z}_0'$ (up to very small accuracy errors). The reason for the transformation and the characteristic impedance eigenvalues to be frequency independent is the frequency independent fixed value of $\epsilon_r = 2.1$ we used in our electromagnetic simulations.

We also want to mention that dealing with lossless MTL, $\boldsymbol{T}_V$ and $\boldsymbol{Z}_0'$ must be real. Because we use results from an electromagnetic simulation, which certainly has small errors, we get very small imaginary values, which we eliminate by taking the real part.

The characteristic impedance eigenvalues Z_0' obtained are: 424.13, 183.87 and 87.71Ω, and the relative (equivalent) dielectric constant ϵ_{eq} come out: 1.7713, 1.9953 and 2.0857 for modes 1,2 and 3 respectively. It it interesting to remark that if we transform the eigenvalues matrix $\boldsymbol{Z}_0'$ scaled by $\sqrt{\epsilon_{eq}}$, by $\boldsymbol{T}_V Z_0' \sqrt{\epsilon_{eq}} \boldsymbol{T}_V^T$ (see Eq. (38)), we recover with good accuracy the homogeneous $\boldsymbol{Z}_0$ matrix in Eq. (77).

So using Eq. (68) we get

$$\boldsymbol{G}_{pre} = \begin{pmatrix} 2.3146 & 0.00171 & 0.56562 \\ 1.6355 & 1.3573 & -0.51086 \\ 0.67446 & 1.3538 & 1.0818 \end{pmatrix}. \tag{79}$$

and $\boldsymbol{G}_{post} = \boldsymbol{G}_{pre}^{-1}$, as explained before. This matrix does not match the transformers' block in Figure 5, which has been designed for the homogeneous case, but except of this the figure is valid, and we do not issue another figure.

In table IV we show the results obtained from the Designer simulation and the analytical results with the processing blocks. One observes that the crosstalk obtained by Designer or analytical calculation is not 0, but very small. This is because of the electromagnetic inaccuracies, discussed before, and this is not a fundamental issue, but rather an accuracy issue which can be improved. From the diagonalization, one obtains ϵ_{eq} from Eq. (49) for the modes, which come out 1.7713, 1.9953 and 2.0857 for modes 1,2 and 3 respectively. Because the signal on conductor i has been mapped to mode i by the $\boldsymbol{T}_V$ transformation, the 50, 100 and 200MHz, "feel" relative dielectric constants of 1.7713, 1.9953 and 2.0857, respectively. Therefore the delay angles of 15.972^o, 33.901^o and 69.323^o, according to Eq. (49) appearing in table IV.

TABLE IV: Voltages V_{in} and V_{out} measured by Designer, and calculated analytically with processing units, for the non homogeneous case. Magnitude is in [V], phase is in degrees and the upper row shows the frequencies in [MHz].

f	Simulated			Calculated		
	50	100	200	50	100	200
$V_{in\,1}$	1 ∠0	0.14m ∠-79.4	0.54m ∠-136.2	1 ∠0	0.16m ∠-88.6	0.49m ∠-142.1
$V_{in\,2}$	0.059m∠-81.5	1 ∠0	1.1m ∠-146.9	0.044m∠-66.3	1 ∠0	1.39m ∠-151.7
$V_{in\,3}$	0.15m ∠-100.4	0.59m ∠-119.4	1 ∠0.03	0.12m ∠-98.6	0.59m ∠-119.5	1 ∠0
$V_{out\,1}$	1 ∠-15.97	0.035m∠-56.2	0.27m ∠90.1	1 ∠-15.97	0.017m∠-47	0.24m ∠120.6
$V_{out\,2}$	0.019m∠73.2	1 ∠-33.9	0.032m∠24.8	0.008m∠-9.6	1 ∠-33.9	0.026m∠-116.3
$V_{out\,3}$	0.022m∠116.8	0.018m∠58.1	1 ∠-69.3	0.012m∠138.2	0.011m∠ 4.53	1 ∠-69.3

VIII. CONCLUSION

We showed in this work that it is possible to send the maximum number of signals (N) through a lossless MTL of $N + 1$ conductors, without any crosstalk or return loss, using a pre-processing unit at the input of the MTL and a post-processing unit at the output of the MTL.

In the lossy case the pre and post-processing units would be complex, with frequency dependent coefficients, but for low losses, the imaginary part being small, we believe that the method can still be efficient in a frequency range of interest.

The processing units have been implemented in our simulations by networks of transformers and further research is needed for deriving a good practical implementation for those processing units.

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